10.1 - Graphs and Graph Models

Introduction

Definition 1
A graph \( G = (V, E) \) consists of \( V \), a nonempty set of vertices (or nodes) and \( E \), a set of edges. Each edge has either one or two vertices associated with it, called its endpoints. An edge is said to connect its endpoints.

The set of vertices \( V \) of a graph \( G \) may be infinite. A graph with an infinite vertex set or an infinite number of edges is called an infinite graph. A graph with a finite vertex set and a finite edge set is called a finite graph.

A graph in which each edge connects two different vertices and where no two edges connect the same pair of vertices is called a simple graph. Graphs with multiple edges connecting the same vertices are called multigraphs.

An edge that connects a vertex to itself is called a loop. Graphs that include loops and possibly multiple edges connecting the same pair of vertices are called pseudographs.

To this point, the graphs discussed have been examples of undirected graphs and their edges are said to be undirected. In an undirected graph, an edge between vertices \( u \) and \( v \) is denoted by \( \{u, v\} \). In a multigraph, if \( m \) different edges are associate with the pair of vertices \( \{u, v\} \), then we say that \( \{u, v\} \) is an edge with multiplicity \( m \).

Definition 2
A directed graph (or digraph) \( (V, E) \) consists of a nonempty set of vertices \( V \) and a set of directed edges (or arcs) \( E \). Each directed edge is associated with an ordered pair of vertices. The directed edge associated with the ordered pair \( (u, v) \) is said to start at \( u \) and end at \( v \).

If a directed graph has no loops and has no multiple directed edges, it is called a simple directed graph. Directed graphs may have multiple directed edges from a vertex to a second (possibly the same) vertex. Such graphs are called directed multigraphs. When there are \( m \) directed edges associate to an ordered pair of vertices \( (u, v) \), we way that \( (u, v) \) is an edge of edge with multiplicity \( m \). A graph with both directed and undirected edges is called a mixed graph.
When classifying graphs, there are 3 questions to consider:
1. Are the edges of the graph directed or undirected (or both)?
2. If the graph is undirected, are multiple edges present that connect the same pair of vertices?
3. Are loops present?

<table>
<thead>
<tr>
<th>TABLE 1 – Graph Terminology</th>
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<tbody>
<tr>
<td>Type</td>
</tr>
<tr>
<td>Simple graph</td>
</tr>
<tr>
<td>Multigraph</td>
</tr>
<tr>
<td>Pseudograph</td>
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<tr>
<td>Simple directed graph</td>
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<tr>
<td>Directed multigraph</td>
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<tr>
<td>Mixed graph</td>
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</tbody>
</table>

**Example 10.1.1**

Determine whether the graph shown has directed or undirected edges, whether it has multiple edges, and whether it has one or more loops. Use your answers to determine the type of graph in Table 1 this graph is.
Graph Models

Graphs have a wide variety of applications. These include:

- Social Networks
- Communication Networks
- Information Networks
- Transportation Networks
- Software Design Networks
- Biological Networks
- Tournaments