FINAL EXAM REVIEW ANSWERS

1. **a)** 0 **b)** 0 **c)** 0 **d)** 2 **e)** No

f) –2 **g)** 1 **h)** DNE **i)** –2 **j)** No

k) 0 **l)** 0 **m)** 0 **n)** 0 **o)** Yes

2. **a)** DNE (left and right limits don't match)
b) No
c) 7
d) Yes

3. **a)** –5 ($\frac{0}{0}$ form – use L'H once)
b) $^{6}/_{13}$
c) 3 ($\frac{0}{0}$ form – use L'H twice)
d) ∞ ($\frac{\infty}{\infty}$ form – use L'H 3 times)
e) 2 ($\frac{\infty}{\infty}$ form – use L'H once)

4. Point is (25, 10), slope is $^{1}/_{5}$ → $y = {}^{1}/_{5}x + 5$

5. Point is (2, 26), slope is 29 → $y = 29x - 32$

6. **a)** $f'(x) = \dfrac{x^2 + 6x - 12}{(x+3)^2}$ **b)** $f'(x) = 4e^{2x}\left(2x^2 - 12x - 7\right)$

c) $f'(x) = \dfrac{15x^2 - 2}{5x^3 - 2x}$ **d)** $f'(x) = 16e^{4x^4 + 16x}(x^3 + 1)$

e) $f'(x) = 7xe^{-3x}\left(-3x + 2\right)$ **f)** $f'(x) = \dfrac{1 - \frac{3}{x} - \ln(3x)}{(x-3)^2}$

7. $\dfrac{dy}{dx} = \dfrac{7x - 2y}{5y + 2x}$

8. $\dfrac{dy}{dx} = \dfrac{3x^2 - 2xe^y}{x^2e^y + 1}$

9. **a)** $\dfrac{(5x-2)^7}{35} + C$ **b)** $\frac{1}{21}\left(7x^2\right)^{\frac{3}{2}} + C$

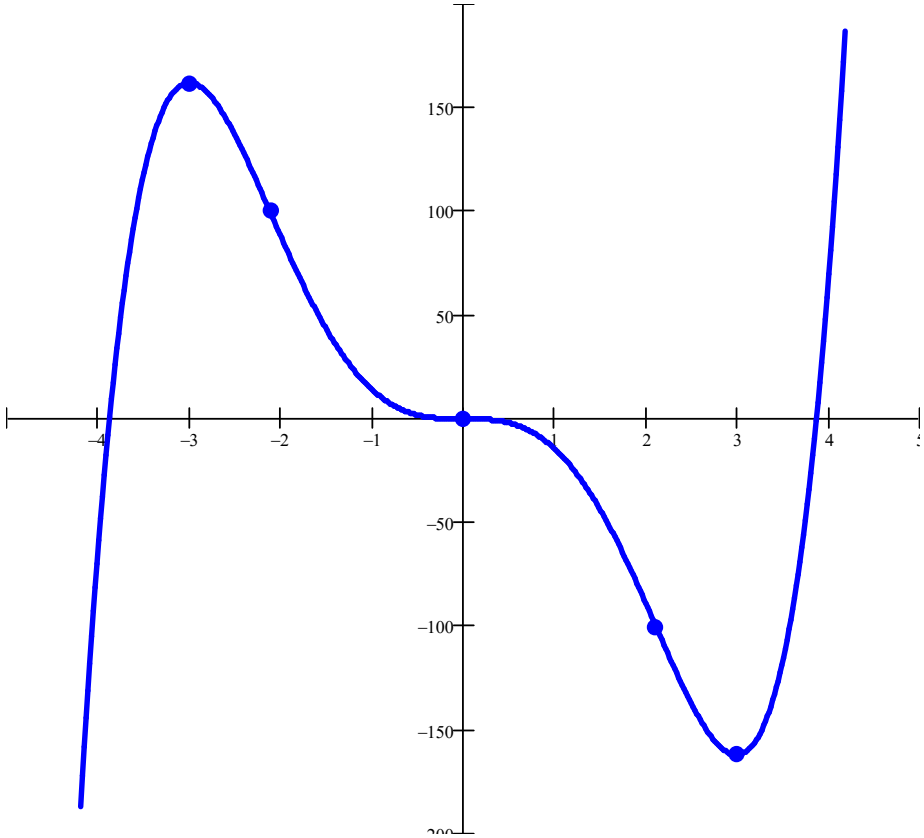
c) $\frac{2}{9}\ln\left|3x^3 - 8\right| + C$ **d)** 12

e) $\frac{1}{2}e^{10} - \frac{1}{2}e \approx 11011.87$ **f)** ¼

10. –11

11. a)

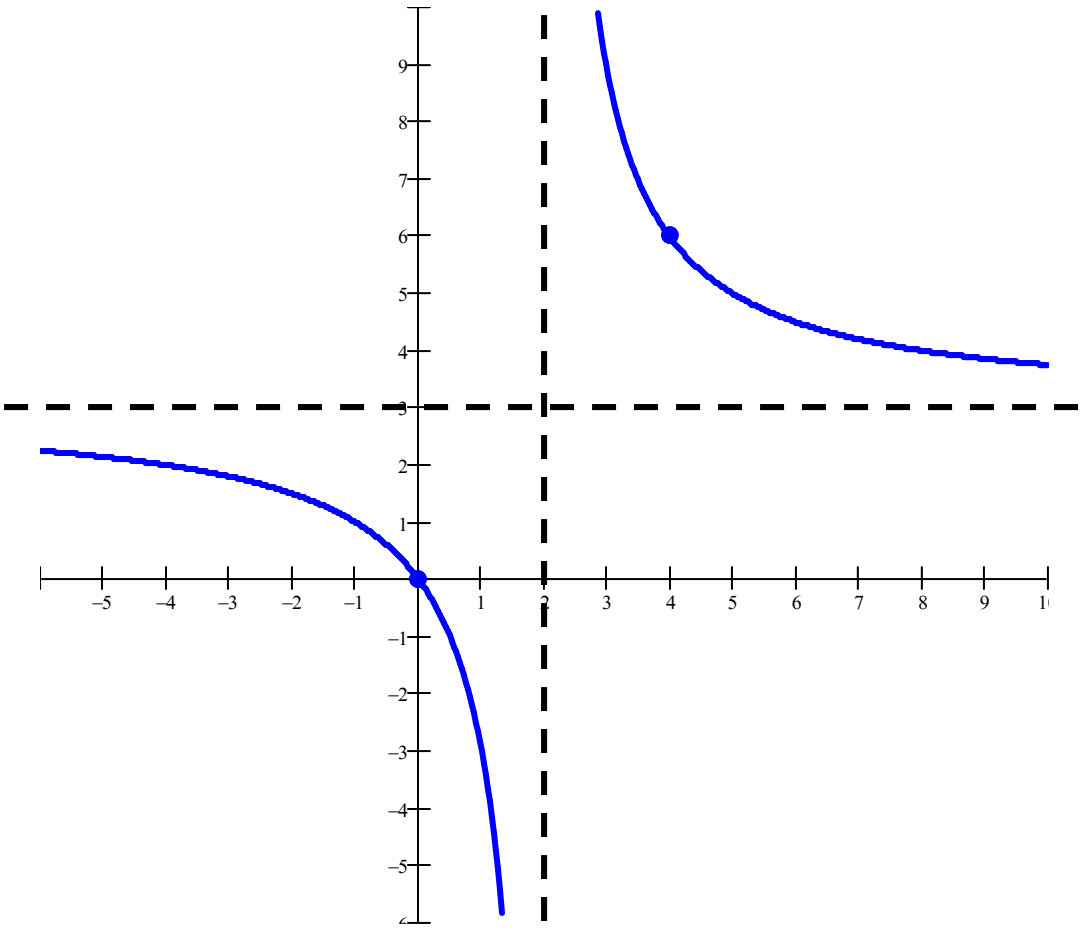
First Derivative Test	Second Derivative Test
$f'(x) = 5x^4 - 45x^2$ $C.V.\ at\ x = 0, \pm 3$ + - - + ←--------|--------|---------|----------→ -3 0 3 Increasing: $(-\infty, -3) \cup (3, \infty)$ Decreasing: $(-3, 0) \cup (0, 3)$ Maximum: (–3, 162) Minimum: (3, –162)	$f''(x) = 20x^3 - 90x$ $C.V.\ at\ x = 0, \pm\sqrt{\frac{9}{2}}$ - + - + ←--------|--------|---------|----------→ $-\sqrt{\frac{9}{2}}$ 0 $\sqrt{\frac{9}{2}}$ Concave Down: $\left(-\infty, -\sqrt{\frac{9}{2}}\right) \cup \left(0, \sqrt{\frac{9}{2}}\right)$ Concave Up: $\left(-\sqrt{\frac{9}{2}}, 0\right) \cup \left(\sqrt{\frac{9}{2}}, \infty\right)$ Inflection Points: $\left(-\sqrt{\frac{9}{2}}, 100.23\right)$, $(0,0)$, $\left(\sqrt{\frac{9}{2}}, -100.23\right)$



11. **b)** Note that this graph has a vertical asymptote at x = 2 and a horizontal asymptote at y = 3 (limit of f(x) as x approaches infinity)

First Derivative Test	Second Derivative Test
$f'(x) = \frac{-6}{(x-2)^2} = -6(x-2)^{-2}$ *C.V. at* $x = 2$ (*vertical asymptote*) - - ←----------|----------→ 2 Decreasing: $(-\infty, 2) \cup (2, \infty)$ No Maximum or Minimum	$f''(x) = \frac{12}{(x-2)^3} = 12(x-2)^{-3}$ *C.V. at* $x = 2$ (*vertical asymptote*) - + ←----------|----------→ 2 Concave Down: $(-\infty, 2)$ Concave Up: $(2, \infty)$ No Inflection Points

Plot a couple of random points in each section to get a good sketch.



12. **a)** $R'(x) = 16 + 6x$

b) $R'(200) = \$1216$

For every extra $1 spent on advertising, revenue will increase by $1216.

13. **a)** $P(x) = R(x) - C(x) = 6x^2 - 1100x - 500$

b) $R'(x) = 20x - 1000 \qquad C'(x) = 8x + 100 \qquad P'(x) = 12x - 1100$

14. $s'(t) = -32t + 40 \qquad set\ \ s'(t) = 0$

Max height occurs when t = 1.25 seconds.

s(1.25) = 28 feet

15. $S'(x) = -3x^2 + 6x + 360$, Critical values at x = –10 & 12 (–10 not in the given interval)

S(6) = 7052

S(12) = 8024 → maximum occurs at a temperature of 12 degrees Celsius

S(20) = 5400

16. $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$ Given: $^{dr}/_{dt} = -0.2$, r = 4.

$\frac{dV}{dt} \approx -40.2 \frac{cm^3}{hr}$ (decreasing)

17. **a)** $y = 2x^2 - x^3 + C \ \rightarrow\ C = 6 \ \rightarrow\ y = 2x^2 - x^3 + 6$

b) $\ln|y| = \frac{-1}{x} + C \ \rightarrow\ C = 1 \ \rightarrow\ \ln|y| = \frac{-1}{x} + 1 \qquad \left(or\ \ y = e^{\frac{-1}{x}+1}\right)$