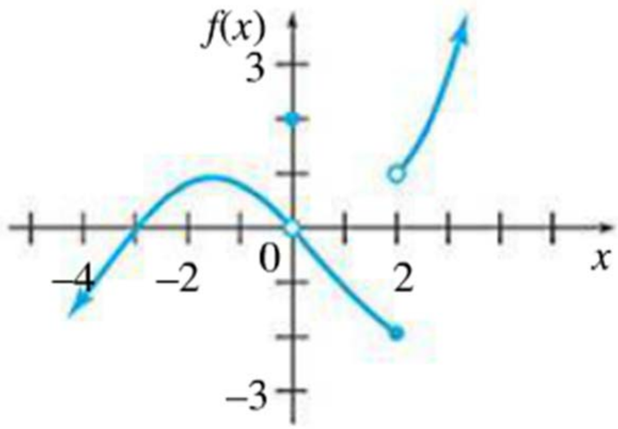
BUSINESS CALCULUS – FINAL EXAM REVIEW

1. Use the following graph to answer parts (a) – (o)



a) $\lim\limits_{x \to 0^-} f(x) =$ **b)** $\lim\limits_{x \to 0^+} f(x) =$

c) $\lim\limits_{x \to 0} f(x) =$ **d)** $f(0) =$

e) Is the graph continuous at x = 0?

f) $\lim\limits_{x \to 2^-} f(x) =$ **g)** $\lim\limits_{x \to 2^+} f(x) =$

h) $\lim\limits_{x \to 2} f(x) =$ **i)** $f(2) =$

j) Is the graph continuous at x = 2?

k) $\lim\limits_{x \to -3^-} f(x) =$ **l)** $\lim\limits_{x \to -3^+} f(x) =$

m) $\lim\limits_{x \to -3} f(x) =$ **n)** $f(-3) =$

o) Is the graph continuous at x = –3?

2. Let $f(x) = \begin{cases} x+5, & \text{if } x < 0 \\ x^2 - 2, & \text{if } 0 \le x \le 3 \\ 10 - x, & \text{if } x > 3 \end{cases}$.

a) $\lim\limits_{x \to 0} f(x) =$

b) Is f(x) continuous at x = 0?

c) $\lim\limits_{x \to 3} f(x) =$

d) Is f(x) continuous at x = 3?

3. Calculate the following limits. Use L'Hospital's Rule when appropriate.

a) $\lim\limits_{x \to -2} \dfrac{x^2 - x - 6}{x + 2} =$ **b)** $\lim\limits_{x \to 2} \dfrac{3x}{7x - 1} =$

c) $\lim\limits_{x \to 0} \dfrac{5x^3 + 3x^2}{x^2} =$ **d)** $\lim\limits_{x \to \infty} \dfrac{e^x}{x^3 + 2x} =$

e) $\lim\limits_{x \to \infty} \dfrac{8x + 2}{4x - 5} =$

4. Write the equation of the tangent line to $f(x) = 2\sqrt{x}$ at x = 25.

5. Write the equation of the tangent line to $f(x) = 2x^3 + 5x$ at x = 2.

6. Calculate the following derivatives:

a) $f(x) = \dfrac{x^2 - 4x}{x+3}$ (use the Quotient Rule and simplify as much as possible)

b) $f(x) = \left(4e^{2x}\right)\left(x^2 - 7x\right)$ (use the Product Rule and simplify as much as possible)

c) $f(x) = \ln(5x^3 - 2x)$

d) $f(x) = e^{4x^4 + 16x}$

e) $f(x) = 7x^2 e^{-3x}$

f) $f(x) = \dfrac{\ln(3x)}{x-3}$

7. Find the derivative using Implicit Differentiation: $7x^2 = 5y^2 + 4xy + 1$

8. Find the derivative using Implicit Differentiation: $x^2 e^y + y = x^3$

9. Calculate the following integrals:

a) $\int (5x-2)^6 \, dx$

b) $\int x\sqrt{7x^2} \, dx$

c) $\int \dfrac{2x^2}{3x^3 - 8} \, dx$

d) $\int_{-2}^{2} (4x+3) \, dx$

e) $\int_{0}^{3} x e^{x^2+1} \, dx$

f) $\int_{1}^{3} \dfrac{1}{(x-5)^2} \, dx$

10. Calculate the average value of $f(x) = 2 - 3x^2$ over the interval [1, 3].

11. Use the First and Second Derivative test to determine the open intervals where f(x) is increasing, decreasing, concave up, concave down. Find the critical values, relative max/min and inflection points. Sketch a graph.

a) $f(x) = x^5 - 15x^3$

b) $f(x) = \dfrac{3x}{x-2}$

12. A company has found that its revenue is related to advertising expenditures by the function $R(x) = 5000 + 16x + 3x^2$ where R(x) is the revenue in dollars when x dollars are spent on advertising.

a) Find the marginal revenue function.

b) Calculate the marginal revenue when $200 is spent on advertising. Interpret this value using a complete sentence.

13. Suppose that the cost and revenue functions for a brewery are given by: $C(x) = 4x^2 + 100x + 500$ *and* $R(x) = 10x^2 - 1000x$ where x is the number of thousands of barrels produced.

a) Find the profit function.
b) Find the marginal cost, marginal revenue, and marginal profit functions.

14. Two Atlantic Institute of Technology senior physics majors determined that when a bottle of French champagne is shaken several times, held upright, and uncorked, its cork travels according to the function $s(t) = -16t^2 + 40t + 3$ where s is its height (in feet) above the ground t seconds after being released. What is the maximum height achieved by the cork?

15. The number of salmon swimming upstream to spawn is approximated by $S(x) = -x^3 + 3x^2 + 360x + 5000, \quad 6 \le x \le 20$, where x represents the temperature of the water in degrees Celsius. Find the water temperature that produces the maximum number of salmon swimming upstream.

16. A spherical snowball is melting. The radius of the snowball is decreasing at a rate of 0.2 cm per hour. At what rate is the volume of the snowball changing at the instant the radius of the snowball is 4 cm? Is the volume of the snowball increasing or decreasing?

$$V = \frac{4}{3}\pi r^3$$

17. Find the particular solution for each initial value problem.

a) $\frac{dy}{dx} + 3x^2 = 4x \qquad y(0) = 6$

b) $x^2 \frac{dy}{dx} = y \qquad y(1) = -1$