MATH 1324 Final Exam Review Sheet (Chapters 1 – 8)

Name: _____ Date: _____

Directions: **You must show all your work, all the time, no exceptions, in pencil, to receive full credit.**
Remember: ***I cannot grade your brain, nor can I grade your calculator!***

CH.1 & 2

1. Of the equations $p + 6q = 420$ and $p = 6q + 60$, one is the supply function for a product and one is the demand function for that same product. Find the market equilibrium. Do so algebraically. *Show all work.*

2. A certain commodity has the following costs for a period of time: Fixed Costs = \$1,500 and Variable Costs per Unit = \$22. The commodity was sold for \$52 per unit. What is the total cost function? What is the total revenue function? What is the total profit function? What is the break-even quantity and price? Do so algebraically. *Show all work.*

3. The profit function is given by $P(x) = 82x - 0.10x^2 - 1600$, where x is the number of units produced and sold. Break-even points will occur at all values of x where $P(x) = 0$. How many units will give a break-even point for the product? Do so algebraically. *Show all work.*

4. Suppose the supply function for a product is $p = 0.1q^2 + 1$ and the demand function is $p = 85 - 0.2q + 0.1q^2$. What is the market equilibrium point? Do so algebraically. *Show all work.*

5. Suppose the supply function for a product is $p = q^2 + 300$ and the demand function is $p + q = 410$. Find the equilibrium quantity and price. Do so algebraically. *Show all work.*

6. If total costs for a product are given by $C(x) = 1760 + 8x + 0.6x^2$ and total revenues are given by $R(x) = 100x - 0.4x^2$, find the break-even quantities. Do so algebraically. *Show all work.*

7. Find the maximum revenue for $C(x) = 900 + 25x$ and $R(x) = 100x - x^2$. Do so algebraically. *Show all work.*

8. Given $C(x) = 360 + 10x + 0.2x^2$ and $R(x) = 50x - 0.2x^2$, find the level of production that gives maximum profit and find the maximum profit. Do so algebraically. *Show all work.*

CH.3

Using Gauss-Jordan Elimination, solve each system using matrices. *Show all work.*

9. $\begin{cases} x + y + z = 4 \\ 3x + 4y - z = -1 \\ 2x - y + 3z = 3 \end{cases}$

10. $\begin{cases} x + y - 2z = 5 \\ 3x + 2y + 5z = 10 \\ -2x - 3y + 15z = 2 \end{cases}$

11. $\begin{cases} x - 3y + z = 4 \\ 2x - 5y - z = 6 \end{cases}$

In each system of equations, solve using inverse matrices. *Show all work.*

12. $\begin{cases} x + 2z = 5 \\ 3x + 4y - z = 2 \\ x + y = -3 \end{cases}$

13. $\begin{cases} 3x + 3y + 2z = 1 \\ -x + 4y + 2z = -10 \\ 2x + 5y + 3z = -6 \end{cases}$

CH.4

LOOK AT #3 ON TEST#3!!!

In problems 14 – 15, **use the simplex method** to solve the linear programming problems. Use the ROWOPS program on your calculator to obtain the final matrix. *Show all work—including the ROWOPS you used in your graphing calculator.*

14. Maximize $f = 7x + 12y$ subject to $\begin{cases} 7x + 3y \le 105 \\ 2x + 5y \le 59 \\ x + 7y \le 70 \end{cases}$

15. Maximize $f = 3x + 4y$ subject to $\begin{cases} x + 4y \le 160 \\ x + 2y \le 100 \\ 4x + 3y \le 300 \end{cases}$

CH.5

Solve the following word problem. Do so algebraically. *Show all work.*

16. If $1000 is invested at 12%, compounded monthly, the future value S at any time t (years) is given by the function: $S(t) = 1000(1.01)^{12t}$. How long will it take for the amount to double? *Round to the nearest whole year.*

CH.6

FORMULAS:

Simple Interest: $I = Prt$

Future Value of Investment or Loan: $A = P + I$

Future Value (Annual Compounding): $A = P(1 + r)^t$

Future Value (Periodic Compounding): $A = P\left(1 + \frac{r}{n}\right)^{nt}$

Future Value (Compounding Continuously): $A = Pe^{rt}$

Annual Percentage Yield (APY* for Periodic Compounding): $APY = \left(1 + \frac{r}{n}\right)^{nt} - 1$

Annual Percentage Yield (APY* for Compounding Continuously): $APY = e^{rt} - 1$

APY is always represented as a decimal.

Future Value of an Ordinary Annuity: $A = \frac{P\left[\left(1 + \frac{r}{n}\right)^{nt} - 1\right]}{\frac{r}{n}}$

Future Value of an Annuity Due: $A = P\left[\frac{\left(1+\frac{r}{n}\right)^{nt} - 1}{\frac{r}{n}}\right]\left(1 + \frac{r}{n}\right)$

Present Value of an Ordinary Annuity: $A = \frac{P\left[1 - \left(1 + \frac{r}{n}\right)^{-nt}\right]}{\frac{r}{n}}$

Present Value of an Annuity Due: $A = P\left[\frac{1 - \left(1+\frac{r}{n}\right)^{-nt}}{\frac{r}{n}}\right]\left(1 + \frac{r}{n}\right)$

Present Value of a Deferred Annuity Due: $A = P\left[\frac{1 - \left(1+\frac{r}{n}\right)^{-nt}}{\frac{r}{n}}\right]\left(1 + \frac{r}{n}\right)^{-k}$

Periodic Payments of an Ordinary Annuity for an Amortization Schedule: $P = A\left[\frac{\frac{r}{n}}{1-\left(1+\frac{r}{n}\right)^{-nt}}\right]$

$$Unpaid\ Balance = P\left[\frac{1-\left(1+\frac{r}{n}\right)^{-(n-k)}}{\frac{r}{n}}\right]$$

CH.7
FORMULAS:

$$P(E) = \frac{n(E)}{n(S)}$$ ← Number of elements in the event (**SUCCESS**)
← Number of elements in the sample space (**TOTAL**)

If *P* (*E*) = 0, then the event *E* is called an **impossible event**.

If *P* (*E*) = 1, then the event *E* is called a **certain event**.

Complementary Events: $P(E)+P(\bar{E})=1$

Union of Two Events

Probability of (A **or** B): *P* (*A* ∪ *B*) = *P* (*A*) + *P* (*B*) – *P* (*A* ∩ *B*)

Note: If *A* and *B* are **mutually exclusive**, then *P* (*A* ∩ *B*) = 0 and we have *P* (*A* ∪ *B*) = *P* (*A*) + *P* (*B*). *These events cannot happen simultaneously because there is **NO INTERSECTION**; one probability has no effect on the other.*

Intersection of Two Events

Probability of (A **and** B): *P* (*A* ∩ *B*) = only where *P* (*A*) **and** *P* (*B*) **overlap** (*simultaneously*)

Probability of **Independent Events:** *P* (*A* and *B*) = *P* (*A*) ·*P* (*B*)

Note: If *A* and *B* are **independent events**, *the occurrence (or nonoccurrence) of o**ne event does not affect the other**. For example, guessing one question on a multiple choice test does not affect the probability that you will (or will not) guess correctly on another question.*

CH.8
FORMULAS:

Mean and Expected Value:

$$\mu = E(x) = \sum(x \cdot P(x)) = x_1 \cdot P(x_1) + x_2 \cdot P(x_2) + x_3 \cdot P(x_3) + x_4 \cdot P(x_4) + \cdots x_n \cdot P(x_n)$$

Expected Value of a Binomial Distribution: $\mu = np$

CH.6

LOOK AT #4 ON TEST#4!!!

17. If $8,000 is borrowed at 12% **simple interest** for 3 years, what is the future value of the loan at the end of 3 years? Do so algebraically. *Show all work.*

18. If $1,000 is invested for 4 years at 8%, **compounded quarterly**, how much interest will be earned? Do so algebraically. *Show all work.*

19. How much money must one invest now in order to have $18,000 in 4 years if the investment earns 5.4%, **compounding monthly**? Do so algebraically. *Show all work.*

20. If an initial investment of $35,000 grows to $257,000 in 15 years, what annual interest rate, **compounding continuously**, was earned? Do so algebraically. *Show all work.*

21. Find the **future value of an ordinary annuity** of $800 paid at the end of every 6 month period for 10 years, if it earns interest at 12%, compounded semiannually. Do so algebraically. *Show all work.*

22. A company wants to have $250,000 available in 4 ½ years for new construction. How much must be **deposited** at the beginning of each quarter to reach this goal if the **investment** earns 10.2%, compounding quarterly? Do so algebraically. *Show all work.*

23. A **debt** of $1,000 with interest at 12% compounded monthly, is amortized by 12 monthly payments of equal size. What is the size of each **payment**? Do so algebraically. *Show all work.*

24. A debt $8,000 is **amortized** with 8 semiannual **payments** of $1,288.29 each. If money is worth 12%, compounded semiannually, find the **unpaid balance** after 5 payments have been made. Do so algebraically. *Show all work.*

25. Complete the next two lines of the **amortization schedule** for a $100,000 loan for 30 years at 7.5%, compounded monthly, with **monthly payments** of $699.22.

Payment Number	Payment Amount	Interest	Balance Reduction	Unpaid Balance
56	$699.22	$594.67	$104.55	$95,042.20
57				
58				

CH.7

26. A card is drawn at random from an ordinary deck of 52 playing cards. What is the **probability** that it is a queen or a jack?

27. A card is drawn from a deck of 52 playing cards. What is the **probability** that it is an ace or a 10?

28. A card is drawn at random from a deck of playing cards. What is the **probability** that it is a king or a red card?

CH.8

29. For the probability distribution shown in the table, find the **expected value** $E(x)$. *Show all work.*

x	P(x)
1	0.4
2	0.3
3	0.2
4	0.1

ANSWER KEY:

1. Market Equilibrium is where Supply equals Demand: $(6q + 60) + 6q = 420$ and solving you obtain $q = 30$ and $p = 240$. Supplying 30 items will demand a price of \$240.

2. Fixed Cost: $\$1500$, Variable Cost: $\$22$ per unit, Revenue: $\$52$ per unit. $C(x) = 22x + 1500,\ R(x) = 52x, P(x) = 30x - 1500, \overline{MC} = 22, \overline{MR} = 52, \overline{MP} = 30$ and the Break-Evan quantity is $x = 50$.

3. Break-Even at x = 20 and x = 800.

4. $q = 420\ and\ p = 17641$

5. $q = 10\ and\ p = 400$

6. Break-Even at x = 64.87 and x = 27.13

7. $x = 50$ and the maximum revenue is \$2500.
8. 50 units for maximum profit. The maximum profit is \$640.

9. $(-3, 3, 4)$

10. No Solution

11. $(-2 + 8z, -2 + 3z, z)$

12. $(-33, 30, 19)$

13. $(4, 5, -13)$
14. The maximum is 168 at $x = 12, y = 7$.

15. The maximum is 260 at $x = 60, y = 20$.

16. $t = \frac{\ln(2)}{12\ln(1.01)} \approx 5.805059741 \approx 6$ years

17. $A = P + I$
$A = P + Prt$
$A = 8000 + 8000(0.12)(3)$
$A = \$\mathbf{10,880.00}$

18. $A = P\left(1 + \frac{r}{n}\right)^{nt}$
$A = 1000\left(1 + \frac{.08}{4}\right)^{4*4}$
$A = \$1372.79$
$I = 1372.79 - \$1000 = \$\mathbf{372.79}$

19. $A = P\left(1 + \frac{r}{n}\right)^{nt}$
$18{,}000 = P\left(1 + \frac{.054}{12}\right)^{12*4}$
$18{,}000 = P(1.240501152)$
$P = 14510.26463$
$P = \$\mathbf{14,510.26}$

20. $A = Pe^{rt}$
$257{,}000 = 35{,}000e^{15r}$
$7.342857143 = e^{15r}$
$\ln(7.342857143) = \ln(e^{15r})$
$\ln(7.342857143) = 15r\ \ln(e)$
$\ln(7.342857143) = 15r$
$\frac{\ln(7.342857143)}{15} = r$
$r = 0.132915202$
$\boldsymbol{r \approx 13.29\%}$

21. $A = 800\left[\frac{\left(1+\frac{.12}{2}\right)^{2*10}-1}{\frac{.12}{2}}\right] \approx \$\mathbf{29,428.47}$

22. $250{,}000 = P\left[\frac{\left(1+\frac{.102}{4}\right)^{18}-1}{\frac{.102}{4}}\right] \approx \$\mathbf{11,117.70}$

23. $P = 1{,}000\left[\frac{\frac{.12}{12}}{1-\left(1+\frac{.12}{12}\right)^{-12}}\right] \approx \$\mathbf{88.85}$

24. $Unpaid\ Balance = 1{,}288.29\left[\frac{1-\left(1+\frac{.12}{2}\right)^{-(8-5)}}{\frac{.12}{2}}\right] \approx \$\mathbf{3,443.61}$

25. Amortization Schedule:

Payment Number	Payment Amount	Interest	Balance Reduction	Unpaid Balance
56	$699.22	$594.67	$104.55	$95,042.20
57	**$699.22**	**$594.01**	**$105.21**	**$94,936.99**
58	**$699.22**	**$593.36**	**$105.86**	**$94,831.13**

26. $P(Queen\ \mathbf{or}\ Jack) = P(Q) + P(J) - P(Queen\ \mathbf{and}\ Jack)$
$P(Q \cup J) = P(Q) + P(J) - P(Q \cap J)$
$P(Q \cup J) = \frac{4}{52} + \frac{4}{52} - 0 = \frac{8}{52} = \mathbf{\frac{2}{13}}$

27. $P(Ace\ \mathbf{or}\ 10) = P(Ace) + P(10) - P(Ace\ \mathbf{and}\ 10)$
$P(A \cup 10) = P(A) + P(10) - P(A \cap 10)$
$P(A \cup 10) = \frac{4}{52} + \frac{4}{52} - 0 = \frac{8}{52} = \mathbf{\frac{2}{13}}$

28. $P(King\ \mathbf{or}\ Red) = P(K) + P(R) - P(King\ \mathbf{and}\ Red)$
$P(K \cup R) = P(K) + P(R) - P(K \cap R)$
$P(K \cup R) = \frac{4}{52} + \frac{26}{52} - \frac{2}{52} = \frac{28}{52} = \mathbf{\frac{7}{13}}$

29. $\mu = E(x) = \sum x \cdot P(x) = 1 \cdot (0.4) + 2 \cdot (0.3) + 3 \cdot (0.2) + 4 \cdot (0.1) = \mathbf{2}$