This problem comes from MATH 2412 Precalculus – David Katz

In Larson’s *Precalculus with Limits*, the author describes a distance \( d \) between a rotating beacon and a wall given by the formula \( d = 4 \tan(2\pi t) \). What physical situation does this problem describe?

The diagram below is from another textbook that describes the same problem and should clarify what relationship the trig formula represents.

The distance between the beacon and the wall is fixed at 4 meters (see below). As the beacon at point A rotates, the path of the light to the wall changes (represented by the dashed lines). So the distance \( d \) from the center of the wall (point R) to the place where the light strikes the wall varies as time passes.

Let’s observe how the distance \( d \) changes for specific values of time \( t \).

When \( t = 0 \) seconds, the beacon is pointing directly at R, and the distance \( d \) from R = 0 because \( d = 4 \tan(2\pi \times 0) = 4\tan(0) = 0 \) meters.

When \( t = 0.10 \) seconds, the beacon is pointing to the right of R, and the distance \( d \) from R to the light \( = 4 \tan(2\pi \times 0.10) \approx 2.91 \) meters.

When \( t = 0.25 \) seconds, the trig function is undefined. This situation corresponds to the beacon pointing in a direction parallel to the wall. As such, the light never strikes the wall, and the distance \( d \) is not defined.

Observe that at the moment immediately before \( t = 0.25 \) seconds, the beacon is pointing very far to the right of R. And at the moment immediately after \( t = 0.25 \) seconds, the beacon is pointing very far to the left of R.

When \( t = 0.30 \) seconds, the beacon is pointing to the left of R, and the distance \( d \) from R to the light \( = 4 \tan(2\pi \times 0.30) \approx -12.31 \) meters. Note that a negative distance means the beacon is pointing to the left of R.

When \( t = 0.50 \) seconds, the beacon has completed its first period of rotation. This fact is clear when we compute the period of this trig function \( \pi/(2\pi) = \frac{1}{2} \). At this point of time, the beacon has returned to pointing directly at R, so again \( d = 0 \) meters.