Gottfried Wilhelm von Leibniz

Born: 1 July 1646 in Leipzig, Saxony (now Germany)
Died: 14 Nov 1716 in Hannover, Hanover (now Germany)

Gottfried Leibniz was the son of Friedrich Leibniz, a professor of moral philosophy at Leipzig. Friedrich Leibniz [3]:-

...was evidently a competent though not original scholar, who devoted his time to his offices and to his family as a pious, Christian father.

Leibniz's mother was Catharina Schmuck, the daughter of a lawyer and Friedrich Leibniz's third wife. However, Friedrich Leibniz died when Leibniz was only six years old and he was brought up by his mother. Certainly Leibniz learnt his moral and religious values from her which would play an important role in his life and philosophy.

At the age of seven, Leibniz entered the Nicolai School in Leipzig. Although he was taught Latin at school, Leibniz had taught himself far more advanced Latin and some Greek by the age of 12. He seems to have been motivated by wanting to read his father's books. As he progressed through school he was taught Aristotle's logic and theory of categorising knowledge. Leibniz was clearly not satisfied with Aristotle's system and began to develop his own ideas on how to improve on it. In later life Leibniz recalled that at this time he was trying to find orderings on logical truths which, although he did not know it at the time, were the ideas behind rigorous mathematical proofs. As well as his school work, Leibniz studied his father's books. In particular he read metaphysics books and theology books from both Catholic and Protestant writers.

In 1661, at the age of fourteen, Leibniz entered the University of Leipzig. It may sound today as if this were a truly exceptionally early age for anyone to enter university, but it is fair to say that by the standards of the time he was quite young but there would be others of a similar age. He studied philosophy, which was well taught at the University of Leipzig, and mathematics which was very poorly taught. Among the other topics which were included in this two year general degree course were rhetoric, Latin, Greek and Hebrew. He graduated with a bachelors degree in 1663 with a thesis *De Principio Individui* (On the Principle of the Individual) which:-

... emphasised the existential value of the individual, who is not to be explained either by matter alone or by form alone but rather by his whole being.

In this there is the beginning of his notion of "monad". Leibniz then went to Jena to spend the summer term of 1663.

At Jena the professor of mathematics was Erhard Weigel but Weigel was also a philosopher and through him Leibniz began to understand the importance of the method of mathematical proof for subjects such as logic and philosophy. Weigel believed that number was the fundamental concept of the universe and his ideas were to have considerable influence of Leibniz. By October 1663 Leibniz was back in Leipzig starting his studies towards a doctorate in law. He was awarded his Master's Degree in philosophy for a dissertation which combined aspects of philosophy and law studying relations in these subjects with mathematical ideas that he had learnt from Weigel. A few days after Leibniz presented his dissertation, his mother died.

After being awarded a bachelor's degree in law, Leibniz worked on his habilitation in philosophy. His work was to be published in 1666 as *Dissertatio de arte combinatoria* (Dissertation on the combinatorial art). In this work

Leibniz aimed to reduce all reasoning and discovery to a combination of basic elements such as numbers, letters, sounds and colours.

Despite his growing reputation and acknowledged scholarship, Leibniz was refused the doctorate in law at Leipzig. It is a little unclear why this happened. It is likely that, as one of the younger candidates and there only being twelve law tutorships available, he would be expected to wait another year. However, there is also a story that the Dean's wife persuaded the Dean to argue against Leibniz, for some unexplained reason. Leibniz was not prepared to accept any delay and he went immediately to the University of Altdorf where he received a doctorate in law in February 1667 for his dissertation *De Casibus Perplexis* (On Perplexing Cases).

Leibniz declined the promise of a chair at Altdorf because he had very different things in view. He served as secretary to the Nuremberg alchemical society for a while (see [187]) then he met Baron Johann Christian von Boineburg. By November 1667 Leibniz was living in Frankfurt, employed by Boineburg. During the next few years Leibniz undertook a variety of different projects, scientific, literary and political. He also continued his law career taking up residence at the courts of Mainz before 1670. One of his tasks there, undertaken for the Elector of Mainz, was to improve the Roman civil law code for Mainz but [3]:-

Leibniz was also occupied by turns as Boineburg's secretary, assistant, librarian, lawyer and advisor, while at the same time a personal friend of the Baron and his family.

Boineburg was a Catholic while Leibniz was a Lutheran but Leibniz had as one of his lifelong aims the reunification of the Christian Churches and [30]:-

... with Boineburg's encouragement, he drafted a number of monographs on religious topics, mostly to do with points at issue between the churches...

Another of Leibniz's lifelong aims was to collate all human knowledge. Certainly he saw his work on Roman civil law as part of this scheme and as another part of this scheme, Leibniz tried to bring the work of the learned societies together to coordinate research. Leibniz began to study motion, and although he had in mind the problem of explaining the results of Wren and Huygens on elastic collisions, he began with abstract ideas of motion. In 1671 he published *Hypothesis Physica Nova* (New Physical Hypothesis). In this work he claimed, as had Kepler, that movement depends on the action of a spirit. He communicated with Oldenburg, the secretary of the Royal Society of London, and dedicated some of his scientific works to the Royal Society and the Paris Academy. Leibniz was also in contact with Carcavi, the Royal Librarian in Paris. As Ross explains in [30]:-

Although Leibniz's interests were clearly developing in a scientific direction, he still hankered after a literary career. All his life he prided himself on his poetry (*mostly Latin*)*, and boasted that he could recite the bulk of Virgil's "Aeneid" by heart. During this time with Boineburg he would have passed for a typical late Renaissance humanist.*

Leibniz wished to visit Paris to make more scientific contacts. He had begun construction of a calculating machine which he hoped would be of interest. He formed a political plan to try to persuade the French to attack Egypt and this proved the means of his visiting Paris. In 1672 Leibniz went to Paris on behalf of Boineburg to try to use his plan to divert Louis XIV from attacking German areas. His first object in Paris was to make contact with the French government but, while waiting for such an opportunity, Leibniz made contact with mathematicians and philosophers there, in particular Arnauld and Malebranche, discussing with Arnauld a variety of topics but particularly church reunification.

In Paris Leibniz studied mathematics and physics under Christiaan Huygens beginning in the autumn of 1672. On Huygens' advice, Leibniz read Saint-Vincent's work on summing series and made some discoveries of his own in this area. Also in the autumn of 1672, Boineburg's son was sent to Paris to study under Leibniz which meant that his financial support was secure. Accompanying Boineburg's son was Boineburg's nephew on a diplomatic mission to try to persuade Louis XIV to set up a peace congress. Boineburg died on 15 December but Leibniz continued to be supported by the Boineburg family.

In January 1673 Leibniz and Boineburg's nephew went to England to try the same peace mission, the French one having failed. Leibniz visited the Royal Society, and demonstrated his incomplete calculating machine. He also talked with Hooke, Boyle and Pell. While explaining his results on series to Pell, he was told that these were to be found in a book by Mouton. The next day he consulted Mouton's book and found that Pell was correct. At the meeting of the Royal Society on 15 February, which Leibniz did not attend, Hooke made some unfavourable comments on Leibniz's calculating machine. Leibniz returned to Paris on hearing that the Elector of Mainz had died. Leibniz realised that his knowledge of mathematics was less than he would have liked so he redoubled his efforts on the subject.

The Royal Society of London elected Leibniz a fellow on 19 April 1673. Leibniz met Ozanam and solved one of his problems. He also met again with Huygens who gave him a reading list including works by Pascal, Fabri, Gregory, Saint-Vincent, Descartes and Sluze. He began to study the geometry of infinitesimals and wrote to Oldenburg at the Royal Society in 1674. Oldenburg replied that Newton and Gregory had found general methods. Leibniz was, however, not in the best of favours with the Royal Society since he had not kept his promise of finishing his mechanical calculating machine. Nor was Oldenburg to know that Leibniz had changed from the rather ordinary mathematician who visited London, into a creative mathematical genius. In August 1675 Tschirnhaus arrived in Paris and he formed a close friendship with Leibniz which proved very mathematically profitable to both.

It was during this period in Paris that Leibniz developed the basic features of his version of the calculus. In 1673 he was still struggling to develop a good notation for his calculus and his first calculations were clumsy. On 21 November 1675 he wrote a manuscript using the $\int f(x)\, dx$ notation for the first time. In the same manuscript the product rule for differentiation is given. By autumn 1676 Leibniz discovered the familiar $d(x^n) = nx^{n-1}dx$ for both integral and fractional n.

Newton wrote a letter to Leibniz, through Oldenburg, which took some time to reach him. The letter listed many of Newton's results but it did not describe his methods. Leibniz replied immediately but Newton, not realising that his letter had taken a long time to reach Leibniz, thought he had had six weeks to work on his reply. Certainly one of the consequences of Newton's letter was that Leibniz realised he must quickly publish a fuller account of his own methods.

Newton wrote a second letter to Leibniz on 24 October 1676 which did not reach Leibniz until June 1677 by which time Leibniz was in Hanover. This second letter, although polite in tone, was clearly written by Newton believing that Leibniz had stolen his methods. In his reply Leibniz gave some details of the principles of his differential calculus including the rule for differentiating a function of a function.

Newton was to claim, with justification, that

..not a single previously unsolved problem was solved ...

by Leibniz's approach but the formalism was to prove vital in the latter development of the calculus. Leibniz never thought of the derivative as a limit. This does not appear until the work of d'Alembert.

Leibniz would have liked to have remained in Paris in the Academy of Sciences, but it was considered that there were already enough foreigners there and so no invitation came. Reluctantly Leibniz accepted a position from the Duke of Hanover, Johann Friedrich, of librarian and of Court Councillor at Hanover. He left Paris in October 1676 making the journey to Hanover via London and Holland. The rest of Leibniz's life, from December 1676 until his death, was spent at Hanover except for the many travels that he made.

His duties at Hanover [30]:-

... as librarian were onerous, but fairly mundane: general administration, purchase of new books and second-hand libraries, and conventional cataloguing.

He undertook a whole collection of other projects however. For example one major project begun in 1678-79 involved draining water from the mines in the Harz mountains. His idea was to use wind power and water power to operate pumps. He designed many different types of windmills, pumps, gears but [3]:-

... every one of these projects ended in failure. Leibniz himself believed that this was because of deliberate obstruction by administrators and technicians, and the worker's fear that technological progress would cost them their jobs.

In 1680 Duke Johann Friedrich died and his brother Ernst August became the new Duke. The Harz project had always been difficult and it failed by 1684. However Leibniz had achieved important scientific results becoming one of the first people to study geology through the observations he compiled for the Harz project. During this work he formed the hypothesis that the Earth was at first molten.

Another of Leibniz's great achievements in mathematics was his development of the binary system of arithmetic. He perfected his system by 1679 but he did not publish anything until 1701 when he sent the paper *Essay d'une nouvelle science des nombres* to the Paris Academy to mark his election to the Academy. Another major mathematical work by Leibniz was his work on determinants which arose from his developing methods to solve systems of linear equations. Although he never published this work in his lifetime, he developed many different approaches to the topic with many different notations being tried out to find the one which was most useful. An unpublished paper dated 22 January 1684 contains very satisfactory notation and results.

Leibniz continued to perfect his metaphysical system in the 1680s attempting to reduce reasoning to an algebra of thought. Leibniz published *Meditationes de Cognitione, Veritate et Ideis* (Reflections on Knowledge, Truth, and Ideas) which clarified his theory of knowledge. In February 1686, Leibniz wrote his *Discours de métaphysique* (Discourse on Metaphysics).

Another major project which Leibniz undertook, this time for Duke Ernst August, was writing the history of the Guelf family, of which the House of Brunswick was a part. He made a lengthy trip to search archives for material on which to base this history, visiting Bavaria, Austria and Italy between November 1687 and June 1690. As always Leibniz took the opportunity to meet with scholars of many different subjects on these journeys. In Florence, for example, he discussed mathematics with Viviani who had been Galileo's last pupil. Although Leibniz published nine large volumes of archival material on the history of the Guelf family, he never wrote the work that was commissioned.

In 1684 Leibniz published details of his differential calculus in *Nova Methodus pro Maximis et Minimis, itemque Tangentibus...* in *Acta Eruditorum*, a journal established in Leipzig two years earlier. The paper contained the familiar d notation, the rules for computing the derivatives of powers, products and quotients. However it contained no proofs and Jacob Bernoulli called it an enigma rather than an explanation.

In 1686 Leibniz published, in *Acta Eruditorum*, a paper dealing with the integral calculus with the first appearance in print of the $\int$ notation.

Newton's *Principia* appeared the following year. Newton's 'method of fluxions' was written in 1671 but Newton failed to get it published and it did not appear in print until John Colson produced an English translation in 1736. This time delay in the publication of Newton's work resulted in a dispute with Leibniz.

Another important piece of mathematical work undertaken by Leibniz was his work on dynamics. He criticised Descartes' ideas of mechanics and examined what are effectively kinetic energy, potential energy and momentum. This work was begun in 1676 but he returned to it at various times, in particular while he was in Rome in 1689. It is clear that while he was in Rome, in addition to working in the Vatican library, Leibniz worked with members of the Accademia. He was elected a member of the Accademia at this time. Also while in Rome he read Newton's *Principia*. His two part treatise *Dynamica* studied abstract dynamics and concrete dynamics and is written in a somewhat similar style to Newton's *Principia*. Ross writes in [30]:-

... although Leibniz was ahead of his time in aiming at a genuine dynamics, it was this very ambition that prevented him from matching the achievement of his rival Newton. ... It was only by simplifying the issues... that Newton succeeded in reducing them to manageable proportions.

Leibniz put much energy into promoting scientific societies. He was involved in moves to set up academies in Berlin, Dresden, Vienna, and St Petersburg. He began a campaign for an academy in Berlin in 1695, he visited Berlin in 1698 as part of his efforts and on another visit in 1700 he finally persuaded Friedrich to found the Brandenburg Society of Sciences on 11 July. Leibniz was appointed its first president, this being an appointment for life. However, the Academy was not particularly successful and only one volume of the proceedings were ever published. It did lead to the creation of the Berlin Academy some years later.

Other attempts by Leibniz to found academies were less successful. He was appointed as Director of a proposed Vienna Academy in 1712 but Leibniz died before the Academy was created. Similarly he did much of the work to prompt the setting up of the St Petersburg Academy, but again it did not come into existence until after his death.

It is no exaggeration to say that Leibniz corresponded with most of the scholars in Europe. He had over 600 correspondents. Among the mathematicians with whom he corresponded was Grandi. The correspondence started in 1703, and later concerned the results obtained by putting $x = 1$ into $1/(1+x) = 1 - x + x^2 - x^3 + ...$ Leibniz also corresponded with Varignon on this paradox. Leibniz discussed logarithms of negative numbers with Johann Bernoulli, see [155].

In 1710 Leibniz published *Théodicée* a philosophical work intended to tackle the problem of evil in a world created by a good God. Leibniz claims that the universe had to be imperfect, otherwise it would not be distinct from God. He then claims that the universe is the best possible without being perfect. Leibniz is aware that this argument looks unlikely - surely a universe in which nobody is killed by floods is better than the present one, but still not perfect. His argument here is that the elimination of natural disasters, for example, would involve such changes to the laws of science that the world would be worse. In 1714 Leibniz wrote *Monadologia* which synthesised the philosophy of his earlier work, the *Théodicée*.

Much of the mathematical activity of Leibniz's last years involved the priority dispute over the invention of the calculus. In 1711 he read the paper by Keill in the *Transactions of the Royal Society of London* which accused Leibniz of plagiarism. Leibniz demanded a retraction saying that he had never heard of the calculus of fluxions until he had read the works of Wallis. Keill replied to Leibniz saying that the two letters from Newton, sent through Oldenburg, had given:-

... pretty plain indications... whence Leibniz derived the principles of that calculus or at least could have derived them.

Leibniz wrote again to the Royal Society asking them to correct the wrong done to him by Keill's claims. In response to this letter the Royal Society set up a committee to pronounce on the priority dispute. It was totally biased, not asking Leibniz to give his version of the events. The report of the committee, finding in favour of Newton, was written by Newton himself and published as *Commercium epistolicum* near the beginning of 1713 but not seen by Leibniz until the autumn of 1714. He learnt of its contents in 1713 in a letter from Johann Bernoulli, reporting on the copy of the work brought from Paris by his nephew Nicolaus(I) Bernoulli. Leibniz published an anonymous pamphlet *Charta volans* setting out his side in which a mistake by Newton in his understanding of second and higher derivatives, spotted by Johann Bernoulli, is used as evidence of Leibniz's case.

The argument continued with Keill who published a reply to *Charta volans*. Leibniz refused to carry on the argument with Keill, saying that he could not reply to an idiot. However, when Newton wrote to him directly, Leibniz did reply and gave a detailed description of his discovery of the differential calculus. From 1715 up until his death Leibniz corresponded with Samuel Clarke, a supporter of Newton, on time, space, freewill, gravitational attraction across a void and other topics, see [4], [62], [108] and [201].

In [2] Leibniz is described as follows:-

Leibniz was a man of medium height with a stoop, broad-shouldered but bandy-legged, as capable of thinking for several days sitting in the same chair as of travelling the roads of Europe summer and winter. He was an indefatigable worker, a universal letter writer (*he had more than* 600 *correspondents*)*, a patriot and cosmopolitan, a great scientist, and one of the most powerful spirits of Western civilisation.*

Ross, in [30], points out that Leibniz's legacy may have not been quite what he had hoped for:-

It is ironical that one so devoted to the cause of mutual understanding should have succeeded only in adding to intellectual chauvinism and dogmatism. There is a similar irony in the fact that he was one of the last great polymaths - not in the frivolous sense of having a wide general knowledge, but in the deeper sense of one who is a citizen of the whole world of intellectual inquiry. He deliberately ignored boundaries between disciplines, and lack of qualifications never deterred him from contributing fresh insights to established specialisms. Indeed, one of the reasons why he was so hostile to universities as institutions was because their faculty structure prevented the cross-fertilisation of ideas which he saw as essential to the advance of knowledge and of wisdom. The irony is that he was himself instrumental in bringing about an era of far greater intellectual and scientific specialism, as technical advances pushed more and more disciplines out of the reach of the intelligent layman and amateur.

Article by: *J J O'Connor* and *E F Robertson*

October 1998