The midpoint of a segment is the point that divides the segment into two congruent pieces. The midpoint of the segment that joins points \( x_1, y_1 \) and \( x_2, y_2 \) is the point \( \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \).

To find the midpoint of the segment joining, \( x_1, y_1 \) and \( x_2, y_2 \), average the two \( x \) values and average the two \( y \) values.

Find the midpoint of the segment with the following endpoints:

Example 1. 5, 8 and 2, 6
Answer:
\[
\left( \frac{5+2}{2}, \frac{8+6}{2} \right) = \left( \frac{7}{2}, \frac{14}{2} \right) = \left( \frac{7}{2}, 7 \right)
\]

Example 2. (-8, 3) and (4, -1)
Answer:
\[
\left( \frac{-8+4}{2}, \frac{3+(-1)}{2} \right) = \left( \frac{-4}{2}, \frac{2}{2} \right) = (-2, 1)
\]

The distance, \( d \), between two points with coordinates \( x_1, y_1 \) and \( x_2, y_2 \) is given by
\[
d = \sqrt{x_2 - x_1}^2 + \sqrt{y_2 - y_1}^2
\]

Find the distance between the given points:

Example 3. (4, -5) and (-2, 3)
\[ d = \sqrt{4 - (-2)^2 + (-5 - 3)^2} \]
\[ d = \sqrt{36 + 64} \]
\[ d = \sqrt{100} \]
\[ d = 10 \]

Find the distance between the given points:

Example 4.  \((5, -3)\) and \((1, 7)\)
\[ d = \sqrt{(5 - 1)^2 + (-3 - 7)^2} \]
\[ d = \sqrt{16 + 100} \]
\[ d = \sqrt{116} \]
\[ d = 2\sqrt{29} \]

For a circle, the midpoint of the segment containing the endpoints of a diameter is the center of the circle. The distance between the endpoints of a diameter is the length of the diameter. The radius is one-half of the diameter.

Example 5.  The given points are the endpoints of a diameter.  Find the center and radius.
\(3, -2\) and \(7, 4\)

To find the center, use the midpoint formula:
\[
\left( \frac{3 + 7}{2}, \frac{-2 + 4}{2} \right)
\]
\[
\left( \frac{10}{2}, \frac{2}{2} \right)
\]
\[(5,1)\]

To find the diameter, use the distance formula:
\[ d = \sqrt{(3 - 7)^2 + (-2 - 4)^2} \]
\[ d = \sqrt{16 + 36} \]
\[ d = \sqrt{52} \]
\[ d = 2\sqrt{13} \]
\[ r = \frac{1}{2} d; \quad r = \frac{2\sqrt{13}}{2} = \sqrt{13} \]

The center of the circle is 5,1 and the radius is \( \sqrt{13} \).
Midpoint and Distance Formula Worksheet

Find the coordinates of the midpoint of the segment joining the given points.

1. (0, 2) and (6, 4)
2. (-2, 2) and (6, 4)
3. (6,-7) and (-6, 3)
4. (-11, 3) and (8,-7)
5. (2.3, 3.7) and (1.5, -2.9)
6. (x, 2) and (x+4, -4)

Find the distance between the two points.

7. (-4, 2) and (2,-1)
8. (-2, -3) and (-2, 4)
9. (3, 2) and (5, -2)
10. (5, -7) and (8,-2)

For the given endpoints of a diameter, find

a. the center of the circle

b. the radius of the circle

11. (-8, 6) and (0, 0)
12. (4,-9) and (-2, -9)
13. (-5, 7) and (4, -2)
14. (-2, -3) and (4, 5)
15. (3, 4) and (2, 1)
Answers to Math 0310 Homework Exercise Supplement – Sect. 13.1

1. (3, 3)
2. (2, 3)
3. (0, -2)
4. \(\left(\frac{-3}{2}, -2\right)\)
5. (1.9, .4)
6. (x+2, -1)
7. \(3\sqrt{5}\)
8. 7
9. \(2\sqrt{5}\)
10. \(\sqrt{34}\)
11. a. (-4, 3) b. \(r = 5\)
12. a. (1, -9) b. \(r = 3\)
13. a. \(\left(\frac{-1}{2}, \frac{5}{2}\right)\) b. \(r = \frac{9\sqrt{2}}{2}\)
14. a. (1, 1) b. \(r = 5\)
15. a. \(\left(\frac{5}{2}, \frac{5}{2}\right)\) b. \(r = \frac{\sqrt{10}}{2}\)