Section 6.1 – Simple Interest and Arithmetic Sequences

Simple Interest

Basic Definitions:

I = Interest in dollars (Can be interest earned on an investment or interest owed on a loan)
P = Principal (Original amount invested or borrowed)
A = Future value
r = annual interest rate in decimal form
t = time invested or term of the loan – in years

	Simple Interest
Interest earned	I = Prt
Future Value (original amount + interest ;What an initial amount P will be worth at a future time)	$A = P(1 + rt)$
Present Value (What amount must be invested now to grow to a amount A)	Solve above for P
Effective Rate (APY)	$r_e = (1 + r/m)^m - 1$

Examples:

1. Find the simple interest if we invest $1974 at 6.3% simple interest for 2 years?

2. Find the future value if we invest $1974 at 6.3% simple interest for 7 months?

3. A person needs to pay off a debt of $7200 that is due in 7 months. How much can he invest now at 5.25% simple interest so that he will have $7200 in 7 months? (We are looking for the “Present Value” here.)

Arithmetic Sequences

Common difference (d) between terms $d = a_n - a_{n-1}$ "second – first" or "third – second", etc.

Example:

2, 5, 8, 11, … d = 3
1, -3, -7, -11, …. d = -4

Note: You can ADD the difference to each term to continue the sequence.

Formula for the ***nth*** term: $a_n = a_1 + (n-1)d$

Examples:

Find a_{40} in the sequence 1, 4, 7, 10, …

a_{40} = 1 + (40-1) d
= 1 +
(39)3 =
118

Find a_{35} in an arithmetic sequence where a_1 = 14 and d = -6

Find a_{155} in the arithmetic sequence 6, 1, -4, -9, …

Consider: Find the sum of the first twelve terms of: **4, 6, 8, …**

The **Sum of *n* Terms of an Arithmetic Sequence** is given by:

$S_n = \frac{n}{2}(a_1 + a_n)$ **or** $S_n = \frac{n}{2}(a_1 + a_1 + d(n-1))$

Examples:

1. Find the sum of the first **fifteen** terms of: **8, 11, 14, ….**

2. Find the sum of the first **twenty** terms of: **10, 9.5, 9, 8.5, …**

Example: Find the sum of: $\mathbf{2 + 5 + 8 + \cdots + 41}$

Example: A display of soup cans has a total of 15 rows. The bottom row has 50 cans, and each successive row has 3 less cans than the prior row.

a) How many soup cans are on the top row?

b) How many total soup cans are in the display?

Section 6.2 – Compound Interest and Geometric Sequences

Basic Definitions:

I = Interest in dollars (Can be interest earned on an investment or interest owed on a loan)
P = Principal   Original amount invested or borrowed
A = Future value
r = annual interest rate in decimal form
t = time invested or term of the loan – in years
m = # compounding periods

m = 1	annual	m = 2	semi-annual
m = 4	quarterly	m = 12	monthly
m = 52	weekly	m = 365	daily

	COMPOUNDING	**CONTINUOUS**
Interest earned	I = A – P	I = A - P
Future Value (original amount + interest; what an initial amount P will be worth at a future time)	$A = P(1 + r/m)^{mt}$	$A = Pe^{rt}$
Present Value (What amount must be invested now to grow to a amount A)	Solve above for P	Solve above for P
Effective Rate (APY)	$r_e = (1 + r/m)^m - 1$	

Examples:

1. Suppose you are investing $6500 at 4.5% interest compounded quarterly for 5 years. Find the future value and the amount of interest that was earned.

2. A couple plan to put a down payment on a house in 5 years. They have an opportunity to invest $10,000 that they have at 5% compounded daily. How much money will they have at the end of 5 years for the down payment?

3. How long will it take \$14,500 to grow to \$25,000 if it is invested at 6.5% compounded monthly?

4. How long will it take \$17,500 to grow to \$25,000 if it is invested at 4.3% compounded continuously?

5. If you want to know which interest rate is better, 5.5% compounded monthly or 6.2% compounded semi-annually, find their effective rates (APR).

Geometric Sequences

A **geometric sequence** is one in which each term after the first is obtained by multiplying the preceding term by a fixed non-zero constant. The constant is called the *common ratio*.

Examples: Write out the first **4** terms of the geometric sequence.

$\{3^{n-1}\}$

Consider: Find the 6^{th} term of: **3, 12, 48, …..**

The ***n^{th}* term of a Geometric Sequence** is given by:

$a_n = a_1 (r)^{n-1}$ **($r \neq 0$)** a_1 = first term, r = common ratio

Examples:

1. Find a formula for the n^{th} term if $\boldsymbol{a_1 = 2}$ and $\boldsymbol{r = 5}$. Then, use that formula to find the 8^{th} term.

2. Find a formula for the n^{th} term of: **5, 15, 45, …..** Then, use that formula to find the 10^{th} term.

3. Find the 6^{th} term of: $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \dots$

The **sum of *n* terms** of a Geometric Sequence is given by:

$$S_n = \frac{a_1\left(1-r^n\right)}{\left(1-r\right)} \qquad (r \neq 0, 1)$$

Examples:

1. Find the sum of the first **twelve** terms of: 3, 6, 12, 24, …

2. Find the sum of the first **seven** terms of: 0.01, 0.1, 1, 10, …..

Section 6.3 – Future Value of Annuities

Ordinary Annuity: A sequence of equal payments made at equal periods of time, where the payments are made at the end of the time period, and where the frequency of payments is the same as the frequency of the compounding of interest.

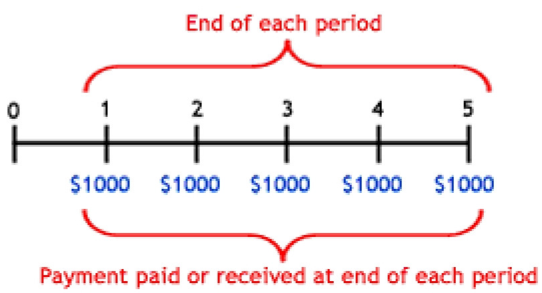
The most common payment frequencies are yearly (once a year), semi-annually (twice a year), quarterly (four times a year) and monthly (once a month). Annuities are used to accumulate money for a goal or to withdraw money on a periodic basis.

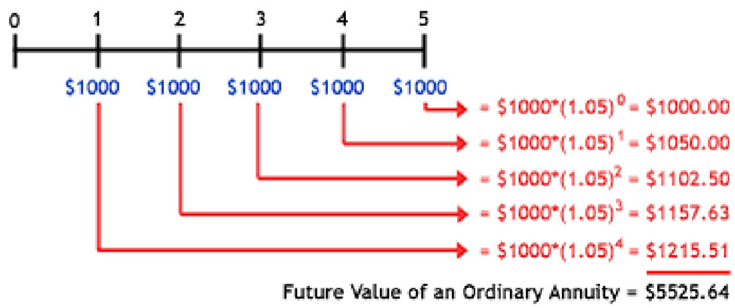
Definitions
S = future value
R = the payment made at the end of each period
i= interest rate per period
n=number of periods

Future value of an ordinary annuity: $S = R\left[\frac{(1+i)^n - 1}{i}\right]$

Present value of an ordinary annuity: $A = R\left[\frac{1-(1+i)^{-n}}{i}\right]$

Ordinary Annuity: Payments are required at the end of each compounding period.





The easy way to calculate this is to use the Future Value of an Annuity formula:

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$

Examples:

Investing $5000 each year for 10 years. Interest is 9% compounded annually. Find the future value of this annuity:

R= ________ i = ____________ (must be per period) n=________ (# of periods)

Investing $300 each quarter for 5 years. Interest is 12% compounded quarterly. Find the future value of this annuity:

R= ________ i = ____________ (must be per period) n=________ (# of periods)

Section 6.4 – Present Value of Annuities

Remember that the previous section dealt with "future value" of an ordinary annuity. That is, if I put money into an annuity for a period of time, what would I have at the end of the time period?

Now we look at the scenario: If I want to receive \$1500 every month for 10 years at retirement, what lump sum must I invest now? This is present value.

You can also use the formula below to figure out what size payments you can get out of a lump sum that you are going to invest.

$$A = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

Important: "i" must be the periodic rate, "n" must be the number of periods, not necessarily the number of years

Examples:

Find the present value of an ordinary annuity of \$5000 paid at the end of each quarter for 6 years, if the interest rate is 6.5% compounded quarterly.

A = ____________ R= _________ i = _____________ (must be per period) n=_________ (# of periods)

Find the present value of an ordinary annuity of \$3000 paid at the end of each 6-month period for 6 years, if the interest rate is 6% compounded semi-annually.

A = ____________ R= _________ i = _____________ (must be per period) n=_________ (# of periods)

Section 6.5 – Loans and Amortization

If you borrow a sum of money and your repayment plan involves making the same size payment each period, this is called **AMORTIZATION**.

From the lender's perspective, they are receiving an annuity from you. You are paying them a fixed amount at the end of a period and the interest is calculated at the end of the period also.

The money the lender lends you initially is like a "Present Value". The amount of your loan payment represents "R". So we use the previously introduced formula for Present Value of an Annuity.

$$A = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

From this, we can answer such questions as:

What **size payments** will I have if I borrow a certain amount of money? (Finding R)

If I know what size payment I can afford, **how much money can I borrow**? (Finding A)

Examples:

John Fare purchased $10,000 worth of equipment by making a $2000 down payment and promising to pay the remainder of the cost in semi-annual payments over the next four years. The interest rate on the debt is 10%, compounding semi-annually.

Find the (a) the size of each payment, (b) the total amount paid over the life of the loan, and (c) the total interest paid over the life of the loan.

A = ____________ R= _________ i = _____________ (must be per period) n=_________ (# of periods)

A woman buys a car for $40,000. If the interest rate on the loan is 12%, compounded monthly, and if she wants to make monthly payments of $700 for 3 years, how much must she have for a down payment?

A = ____________ R= _________ i = _____________ (must be per period) n=_________ (# of periods)

Amortization Schedule **see page 452**

This is just a table that is generated to show how payments are applied. It shows how your loan balance is affected by the payments you make. It can be useful to see how much interest you are paying over the life of the loan, or you can see what you payoff amount would be at a certain time. Typical column headings are shown, but these can have various titles.

Payment #	Payment Amount	Amount Paid To Interest	Amount Paid to Principal	Unpaid Balance	Total Int Paid

Because amortization schedules are typically long, and the calculations are repetitive, these are easily done with software, such as EXCEL.

(ONE LIKE THE TEST)