16.1 Functions and Graphs

**Relation:** A set of ordered pairs.

**Domain:** The set of all $x$-values (first elements) for a relation.

**Range:** The set of all $y$-values (second elements) for a relation.

**Function:** A relation in which every value in the domain is paired with exactly one value in the range.

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**The Vertical Line Test**

To determine whether a relation is a function from its graph, perform a vertical line test:

1. Draw or imagine vertical lines through each point in the domain.
2. If each vertical line intersects the graph at only one point, then the graph is the graph of a function.
3. If any vertical line intersects the graph more than once, then the graph is not the graph of a function.

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Ex. Determine the domain and the range of the relation and determine whether it is a function.

**a)**

![Diagram](image1.png)

- Domain: _________________
- Range: _________________
- Function? ________________

**b)**

![Diagram](image2.png)

- Domain: _________________
- Range: _________________
- Function? ________________
c) Domain: _______________________
Range: _______________________
Function? _______________________

d) \( y = 2 - |x + 1| \) (Window [-6, 4] scale 1 and [-6, 5] scale 1)

\[
\begin{array}{c|c}
   x & Y_1 \\
\hline
   & \\
\end{array}
\]

Domain: _______________________
Range: _______________________
Function? _______________________

e) \{(-1, 0), (3, 0), (2, 6), (4, 5)\}  
\[ \text{Domain: } \quad \text{Range: } \quad \text{Function? } \]
f) \{(-3, 2), (4, 1), (-3, 5), (2, 8)\}  
\[ \text{Domain: } \quad \text{Range: } \quad \text{Function? } \]

g) Top U.S. Last Names

<table>
<thead>
<tr>
<th>Name</th>
<th>% of All Names</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smith</td>
<td>1.006%</td>
</tr>
<tr>
<td>Johnson</td>
<td>0.810%</td>
</tr>
<tr>
<td>Williams</td>
<td>0.699%</td>
</tr>
<tr>
<td>Brown</td>
<td>0.621%</td>
</tr>
<tr>
<td>Jones</td>
<td>0.621%</td>
</tr>
</tbody>
</table>

\[ \text{Domain: } \quad \text{Range: } \quad \text{Function? } \]

Source: Russell Ash, The Top 10 of Everything
Ex. Which of the following equations are functions?

(a) \( y = -x + 1 \)
(b) \( x = -5 \)
(c) \( y = x^2 - 4 \)
(d) \( 2y + 4 = 6 \)

Note: All linear equations are functions except those of the form \( x = a \), which are vertical lines.

❖ FIND THE VALUE OF A FUNCTION

The notation of a function is: \( f(x) \)
read “f of x,” “f at x,” or “the value of f at x”

<table>
<thead>
<tr>
<th>Equation in two variables:</th>
<th>Function notation:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = -4x^2 + 1 )</td>
<td>( f(x) = -4x^2 + 1 )</td>
</tr>
<tr>
<td>( y = \frac{2}{3}x - 5 )</td>
<td>( f(x) = \frac{2}{3}x - 5 )</td>
</tr>
</tbody>
</table>

Ex. Find the function values.

\( f(x) = 2x^2 - 5 \)

(a) \( f(-1) \)      (b) \( f(3) \)      (c) \( f(a) \)

Ex. Given \( g(x) = |2 - x| \), find the values of the function.

(a) \( g(-2) \)      (b) \( g(0) \)      (c) \( g(3) \)