18.3 Absolute-Value Equations and Inequalities

- **Absolute Value**: the distance from zero on a number line.

Ex. \( |x| = 5 \)

The variable can have a value of either 5 or -5 because both of these numbers are 5 units from zero on the number line.

**Absolute Value Property**

If \( |x| = a \), where \( x \) is a variable or an expression and \( a \geq 0 \), then \( x = a \) or \( x = -a \).

Ex. Solve. Write the solution in set notation.

(a) \( |x| = 9 \)   
(b) \( |p| = 0 \)   
(c) \( |x| = -3 \)

In general, the following is true:

- If \( a \) is **positive**, the equation \( |x| = a \) has **two solutions**, \( a \) and \( -a \).
- If \( a \) is **0**, the equation \( |x| = a \) has **one solution**, 0.
- If \( a \) is **negative**, the equation \( |x| = a \) has **no solution**.
Ex. Solve. Write the solution in set notation.

(a)  $|x + 2| = 3$.  

(b)  $|3x| = -8$

(c)  $|5x| - 3 = 37$  

(d)  $3 - 2|x| = -7$

\[ \text{Absolute Value Inequalities} \]

Ex.  $|x| < 5$

The solution set of $|x| < 5$ contains all numbers whose distance from 0 is less than 5 units on the number line.

Ex.  $|x| > 5$

The solution set of $|x| > 5$ contains all numbers whose distance from 0 is more than 5 units on the number line.
Absolute Value Inequalities of the Form $|X| < a$

If $a$ is a positive number, then $|X| < a$ is equivalent to $-a < X < a$, where $X$ is a variable or an expression. This property also holds true for the inequality symbol $\leq$.

Ex. Solve. Write the solution in set-builder notation.

(a) $|y - 2| \leq 5$  
(b) $|x + 1| + 11 < 9$

(c) $-17 + |3 + 5x| < -4$  
(d) $|3x + 6| \leq 0$
**Absolute Value Inequalities of the Form** \(|X| > a\)

If \(a\) is a positive number, then \(|X| > a\) is equivalent to \(X < -a\) or \(X > a\), where \(X\) is a variable or an expression. This property also holds true for the inequality symbol \(\geq\).

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**Ex. Solve. Write the solution in set-builder notation.**

(a) \(|y - 2| \geq 5\)

(b) \(|x + 1| + 11 > 9\)

(c) \(-17 + |3 + 5x| > -4\)

(d) \(|3x + 6| \geq 0\)