19.1 Radical Expressions and Functions

❖ Square Roots

Square Root
The number \( c \) is a square root of \( a \) if \( c^2 = a \).

Ex. Find the square roots of 49.

Principal Square Root
The principal square root of a nonnegative number is its nonnegative square root.

Ex. The principal square root of 16 is 4.

Radical sign: \( \sqrt{\text{ }} \)

Radicand: the expression under the radical sign (ex. \( \sqrt{5} \), \( \sqrt{a} \), \( -\sqrt{3x} \), \( \sqrt{\frac{y^2 + 7}{y}} \))

Index (plural, indices): \( \sqrt[n]{a} \) for \( n \)th root \( \Rightarrow \) the number \( n \) is called the index.

Ex. Identify the radicand and the index for each expression.

(a) \(-7\sqrt{y^2 - 8}\) \hspace{2cm} (b) \(a^2b^5\sqrt[3]{\frac{a}{a^2 - b}}\)

Ex. Simplify

(a) \(\sqrt{196}\) \hspace{2cm} (b) \(-\sqrt[3]{\frac{81}{144}}\) \hspace{2cm} (c) \(\sqrt{0.36}\)

Square roots of perfect square radicands simplify to rational numbers. What happens when we try to simplify a root such as \(\sqrt{11}\) ?

Use a calculator to approximate \(\sqrt{11}\) to 3 decimal places. Check to see that your approximation is reasonable.
Expressions of the Form $\sqrt{a^2}$

Simplifying $\sqrt{a^2}$

For any real number $a$,

$$\sqrt{a^2} = |a|.$$  

(The principal square root of $a^2$ is the absolute value of $a$.)

Ex. Simplify each expression. Assume that the variable can represent any real numbers.

(a) $\sqrt{25t^2}$
(b) $\sqrt{x^2 - 8x + 16}$

(c) $\sqrt{x^{16}}$
(d) $\sqrt{(r - 1)^4}$

Cube Roots

Cube Root

The number $c$ is the cube root of $a$ if $c^3 = a$. In symbols, we write $\sqrt[3]{a}$ to denote the cube root of $a$.

Ex. Simplify (a) $\sqrt[3]{64}$
(b) $-\sqrt[3]{-125y^6}$

Odd and Even $n$th Roots $\sqrt[n]{a}$

Odd Roots
Odd roots of positive numbers are positive.
Odd roots of negative numbers are negative.
Absolute-value signs are not used when finding odd roots.

Even Roots
Even roots of positive numbers are positive.
Negative numbers do not have real $n$th roots when $n$ is even.
Absolute-value signs are often necessary when finding even $n$th roots.
Ex. Simplify. Assume that the variables represent any real numbers. Remember to use absolute-value notation when necessary.

(a) \[ \sqrt[5]{-\frac{32}{243}} \]  
(b) \[ 6\sqrt[5]{(x-5)^6} \]  
(c) \[ 4\sqrt[4]{(-2)^4} \]

Ex. Simplify. Assume that all variables represent nonnegative numbers.

(a) \[ \sqrt[10]{\frac{y^{10}}{9x^6}} \]  
(b) \[ \sqrt[5]{-243d^{15}} \]  
(c) \[ -\frac{4}{\sqrt[n]{16m^4}} \]


\textbf{Radical Functions and Models}

\textbf{Radical Function}: a function that can be describe by a radical expression.

Ex. For \( g(x) = \sqrt{x^2 - 25} \), find the specified function value, if it exists.

(a) \( g(-6) \)  
(b) \( g(3) \)  
(c) \( g(13) \)

Ex. For \( g(x) = -\sqrt[3]{2x-1} \), find the specified function value, if it exists.

(a) \( g(-13) \)  
(b) \( g(63) \)

\textbf{The square-root function}: \( f(x) = \sqrt{x} \)  
Domain: \( [0, \infty) \) or \( \{x \mid x \geq 0\} \)

Graph \( f(x) = \sqrt{x} \).

Ex. Determine the domain of each function described.

(a) \( f(x) = \sqrt[4]{x-7} \)  
(b) \( g(t) = \sqrt[3]{2t-5} \)

(c) \( H(x) = 5 - \sqrt[8]{1-4x} \)  
(d) \( P(x) = 2 + \sqrt{3x-5} \)