4.2 Exponential Functions

**Exponential Functions:** \( f(x) = b^x \) or \( y = b^x \), \( b > 0 \) and \( b \neq 1 \), \( x \in \mathbb{R} \)

◊ **Graphing Exponential Functions**

Ex. Graph each function by making a table or coordinates.

(a) \( f(x) = 3^x \)

\[
\begin{array}{c|c}
 x & y = 3^x \\
-2 & 0.03125 \\
-1 & 0.33333 \\
0 & 1 \\
1 & 3 \\
2 & 9 \\
\end{array}
\]

- **Domain:** __________
- **Range:** __________
- **x-intercept:** __________
- **y-intercept:** __________
- **H.A.:** __________

(b) \( f(x) = \left(\frac{1}{3}\right)^x \)

\[
\begin{array}{c|c}
 x & y = (1/3)^x \\
-2 & 27 \\
-1 & 9 \\
0 & 3 \\
1 & 1 \\
2 & 1/3 \\
\end{array}
\]

- **Domain:** __________
- **Range:** __________
- **x-intercept:** __________
- **y-intercept:** __________
- **H.A.:** __________
(c) \( f(x) = 3^{-x} \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = 3^x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
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<tr>
<td>2</td>
<td></td>
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</tbody>
</table>

Domain: _________________
Range: _________________

\( x \)-intercept: _________________
\( y \)-intercept: _________________

\( H.A. \): _________________

Properties of Exponential Graphs of the Form \( f(x) = b^x \): (p.447)

1) Domain: _________________
   Range: _________________

2) The point that all graphs pass through: _________________
   \( x \)-intercept: _________________
   \( y \)-intercept: _________________

3) \( b > 1 \): \( f(x) = b^x \) is an _________________ exponential function.

4) \( 0 < b < 1 \): \( f(x) = b^x \) is an _________________ exponential function.

5) One-to-One Function; has an inverse function

6) Horizontal Asymptote: _________________

An increasing exponential function is also called an exponential growth function.
A decreasing exponential function is also called an exponential decay function.
Transformations of Exponential Functions

Ex. Given the graph of \( f(x) = 3^x \).

i) Use the transformations of this graph to graph the given function.

ii) Give equations of the asymptotes.

iii) Use the graphs to determine each function’s domain and range.

(a) \( f(x) = 3^x + 2 \)  

(b) \( f(x) = 3^{x-1} \)

H.A.: ________________
Domain: ________________
Range: ________________

H.A.: ________________
Domain: ________________
Range: ________________

The Natural Base \( e \)

\[
\left(1 + \frac{1}{n}\right)^n \approx 2.718281827... \quad \text{as } n \to \infty
\]

\( e \) = irrational number

Natural Exponential Function: \( f(x) = e^x \)

The graph of \( f(x) = e^x \) has the same characteristics as any other exponential functions with base “\( b \)”.

Ex. Evaluate \( f(x) = e^x \) for \( f(\sqrt{7}) \) and \( f(-3) \).

Round to 4 decimal places.
Compound Interest

**Compound Interest:** interest computed on your original investment as well as on any accumulated interest.

**Simple Interest:** \( I = Prt \)

<table>
<thead>
<tr>
<th>Formulas for Compound Interest</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.) <strong>Compound Interest:</strong> Compound interest is paid ( n ) times a year.</td>
</tr>
<tr>
<td>( A = P \left( 1 + \frac{r}{n} \right)^{nt} )</td>
</tr>
<tr>
<td>2.) <strong>Continuous Compounding:</strong> the number of compounding periods increases infinitely.</td>
</tr>
<tr>
<td>( A = Pe^{rt} )</td>
</tr>
</tbody>
</table>

- \( A \): Accumulated amount of money invested after \( t \) years
- \( P \): Principal (original amount invested)
- \( r \): Annual Percentage (Interest) Rate
- \( t \): years
- \( n \): Compounding Periods per year
  - Annually \( n = 1 \)
  - Semi-annually \( n = 2 \)
  - Quarterly \( n = 4 \)
  - Monthly \( n = 12 \)
  - Weekly \( n = 52 \)
  - Daily \( n = 365 \)

Ex. Find the accumulated value of an investment of $5000 for 10 years at an interest rate of 6.5% if the money is

a) compounded quarterly

b) compounded continuously
Ex. (#66) The population of Canada in 2010 was approximately 34 million with an annual growth rate of 0.804%. At this rate, the population $P(t)$ (in Millions) can be approximated by $P(t) = 34(1.00804)^t$, where $t$ is the time in years since 2010. *(Source: www.cia.gov)*

(a) Is the graph of $P$ an increasing or decreasing exponential function?

(b) Evaluate $P(0)$ and interpret its meaning in the context of this problem.

(c) Evaluate $P(5)$ and interpret its meaning in the context of this problem. Round the population value to the nearest million.