8.1 Sequences and Series

**Sequence (Infinite Sequence)** \( \{a_n\} \): a function whose domain is the set of positive integers.

\[ a_n : \text{the } n\text{th term}, \text{ or general terms, of a sequence} \]

\[ a_1, a_2, a_3, a_4, \ldots, a_n, \ldots \]

Ex. Write the first four terms of each sequence whose general term is given.

<table>
<thead>
<tr>
<th>a.) ( a_n = 4n - 1 )</th>
<th>b.) ( c_n = (-1)^{n+1} (n + 4) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_1 = )</td>
<td>( c_1 = )</td>
</tr>
<tr>
<td>( a_2 = )</td>
<td>( c_2 = )</td>
</tr>
<tr>
<td>( a_3 = )</td>
<td>( c_3 = )</td>
</tr>
<tr>
<td>( a_4 = )</td>
<td>( c_4 = )</td>
</tr>
</tbody>
</table>

**Summation Notation**

**Summation Notation**: the sum of the first \( n \) terms of a sequence

\[ \sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + a_4 + \ldots + a_n \]

\( i \): the index of summation \((i, j, \text{ and } k \text{ are used commonly.})\)

\( n \): the upper limit of summation

\( 1 \): the lower limit of summation

Ex. Expand and evaluate the sum:

a.) \( \sum_{k=1}^{4} (k - 3)(k + 2) \)

b.) \( \sum_{i=2}^{4} \left(-\frac{1}{3}\right)^i \)
Ex. Express each sum using summation notation. Use 1 as the lower limit of summation and \( i \) for the index of summation.

a.) \( 1^4 + 2^4 + 3^4 + \cdots + 12^4 \)

b.) \( \frac{1}{9} + \frac{2}{9^2} + \frac{3}{9^3} + \cdots + \frac{n}{9^n} \)

c.) \( \frac{3}{2} + \frac{1}{27} + \frac{5}{32} + \cdots + \frac{n+2}{n^3} \)